COUPLING FINITE ELEMENT AND INTEGRAL EQUATION RI{ PRESENTATIONS TO EFFICIENTLY MODEL LARGE THREE-DIMENSIONAL SCATTERING OBJECTS

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1. INTRODUCI'1ON

"l'he usefulness of finite element modeling follows from the ability to accurately simulate the geometry and three-dimensional fields on the scale of a fraction of a wavelength. To make this modeling practical for engineering design, it is necessary to integrate the stages of geometry modeling and mesh generation, numerical solution and display of fields. "l'he stages of geometry modeling, mesh generation, and field display are commonly completed using commercially available software packages. Algorithms for the numerical solution of the fields need to be written for the specific class of problems considered. Interior problems, i.e. simulating fields in waveguides and cavities, have been successfully solved using finite element methods. Exterior problems, i.e. simulating fields scattered or radiated from structures, are more, difficult to model because of the need to numerically truncate the finite element mesh. To practically compute a solution to exterior problems, the domain must be truncated at some finite surface where the Sommerfeld radiation condition is enforced, either approximately or exactly, Approximate methods attempt to truncate the mesh using only local field information at each grid point, whereas exact methods are global needing information from the entire mesh boundary. This paper outlines a method that couples a three-dimensional finite element solution interior to the bounding surface with an efficient integral equation solution that exactly enforces the Sommerfeld radiation condition [1]. Specifically, edge based, vector finite elements are used to model fields in the interior region,

II. THE INTERIOR FINITE ELEMENT MODEL

The scatterer and surrounding space are broken into two regions--an interior part containing the scatterer and freespace region out to a defined surface, and the exterior homogeneous part (Figure 1). To efficiently model fields in the exterior region, the surface bounding the interior is prescribed to be a *body* of revolution (BOR). In this interior region, the weak form of the wave equation is used to model the geometry and fields

$$\int_V \left[\frac{1}{\epsilon_r} (\nabla \times T) \bullet (\nabla \times H) - k^2 \mu_r T \bullet H \right] dv - j\omega \, \epsilon_o \int_S T \bullet E \times \hat{n} \, ds = 0. \tag{1}$$

II is the magnetic field (the H-equation is used in this paper; a dual E-equation can also be written), T is a testing function, and $E \times \hat{n}$ is the tangential component of E on the BOR surface S. A finite element representation is used to model the fields within this volume. An ensemble of elements filling the interior region, excluding any perfect conducting objects, is created using a
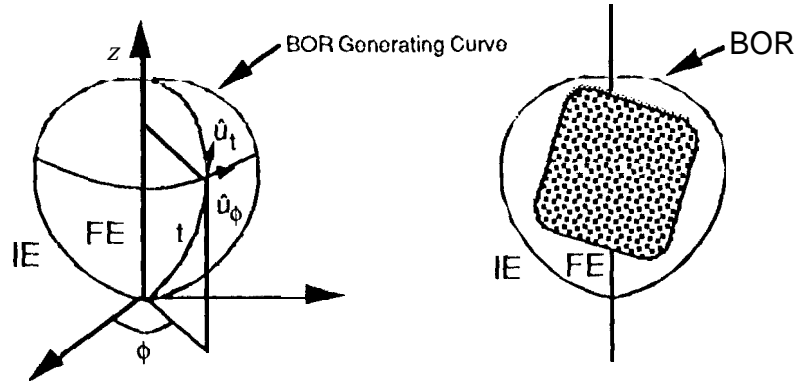
Figure 1. Geometry of scatterer showing interior and exterior regions.

mesh generator. The elements should accurately represent the magnetic field, the geometry of the scatterer, and the bounding BOR surface, Since the scatterer is not a BOR in general, the finite element mesh will extend out from the scatterer to the BOR surface, For an accurate model of the fields, tetrahedral, vector edge elements are used to model $\mathbf{H}$

$$\mathbf{H}(r) = \sum_i h_i \mathbf{W}_i(r) \tag{2}$$

where

$$\mathbf{W}_{mn}(r) = \lambda_m(r)\nabla\lambda_n(r)^- \lambda_n(r)\nabla\lambda_m(r) \tag{2a}$$

and $\lambda(r)$ are the tetrahedral shape functions. Test ing functions are also chosen to be the functions $\mathbf{W(r)}$.

III. AN EFFICIENT EXTERIOR INTEGRAL EQUATION MODEL

In the BOR configuration, a cylindrical coordinate system (p, ϕ, z) is selected for the exterior region, and orthogonal surface coordinates (ϕ, t) are used on the boundary itself, where ϕ is the azimuthal angle variable, while t is the contour length variable along the BOR generating curve.

in the formulation of the integral equation, equivalent electric and magnetic surface currents are defined on the boundary, namely, $\mathbf{J} = \mathbf{n} \times \mathbf{H}, \mathbf{M} = \mathbf{E} \times \mathbf{n}$. These currents produce scattered fields in the exterior region. The sum of scattered and incident fields results in the total field everywhere outside the boundary surface. On the boundary itself, this sum is equal to half the total field. The scattered field in the exterior region is obtained from the tangential currents via an integral over the boundary using a specific free-space Green's function kernel that exploits the rotational symmetry. The electric field integral equation (EFIE) and the magnetic field integral equation (MFIE) are linearly combined resulting in the combined field integral equation (CFIE). This formulation is used to avoid difficulties caused by non-uniqueness in the EFIE and HFIE formulation at interior resonant frequencies. The CFIE is given as

$$\frac{1}{\eta_o} Z_m[\mathbf{M}] + Z_J[\mathbf{J}] = V_i \tag{3}$$

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Operators Z_M and Z_J in (3) are integro-differential operators involving the free space Green's function and $\mathbf{V}_i$ is related to the incident field [2].

IV, COUPLING THE TWO REPRESENTATIONS

Equations for fields in the interior and exterior regions have been specified. It is only necessary to enforce boundary conditions to create a unique solution to Maxwell's equations. Tangential components of $\mathbf{E}$ and $\mathbf{H}$ are enforced to be continuos at the BOR surface by the following two equations

$$\int_s T \bullet (\mathbf{E} \times \hat{n} - \mathbf{M})ds = 0 \tag{4}$$

$$\int_s \hat{n} \times \mathbf{U} \bullet (\hat{n} \times \mathbf{H} - \mathbf{J})ds = 0 \tag{5}$$

where the functions T and U are used as testing functions to enforce continuity in a weak sense. 'l'he first equation (enforcement on $\mathbf{E}$) is substituted into the surface integral of(1). The second condition (on $\mathbf{H}$) is explicity enforced as a separate equation. Together (1), (3) and (5) make up the linear system describing the fields in both regions. It is noted that the Sommerfeld radiation condition is implicitly enforced in (3), and any material boundary conditions are enforced in the finite element representation in (1).

The surface integral in (1) and the first component of the integral in (5) are termed the coupling integrals since they couple interior and exterior field representations. Because the BOR surface is used to separate regions, the standard Fourier series formalism can be applied to express the variation of the unknown surface currents. Additionally, sub-domain basis functions are used to describe current variation along the BOR generating curve. The integral equation basis functions have the form [2]

$$\mathbf{U}_a(t, \ \phi) \ \underset{=}{\frac{T(t)}{p(t)}} e^{jn\phi} \ \hat{a} \tag{6}$$

where 7' is a triangle function and a is either t or ϕ. However, in the interior volume the finite element representation is different, being given by the ensemble of tetrahedral elements; in particular, the union of the boundary finite elements external facets will define the boundary surface of the finite element mesh. It is noted that this surface will generally not coincide with the surface of revolution chosen for the integral equation portion of the problem and only in the limit of fine meshing and generator description will the surfaces come into contact with each other. This difficulty could be removed by introducing isoparametric elements to model the BOR surface S. We are planning to investigate this approach in the future.

V. NUMERICAL RESULTS

The resultant system of equations is solved, giving J, M and 11. Because the system is partitioned, different methods of solution can be used. The finite element block of the matrix system is highly sparse, and the other blocks are banded, The different solution methods will be outlined in a separate paper.

One example calculation is scattering from a homogeneous dielectric sphere of relative permattivity 2. The radius is ka = 1.05, Plotted in Figure 2 are the M_t Fourier components of the surface current as calculated from the above formulation, and CICERO BOR scattering code [2].
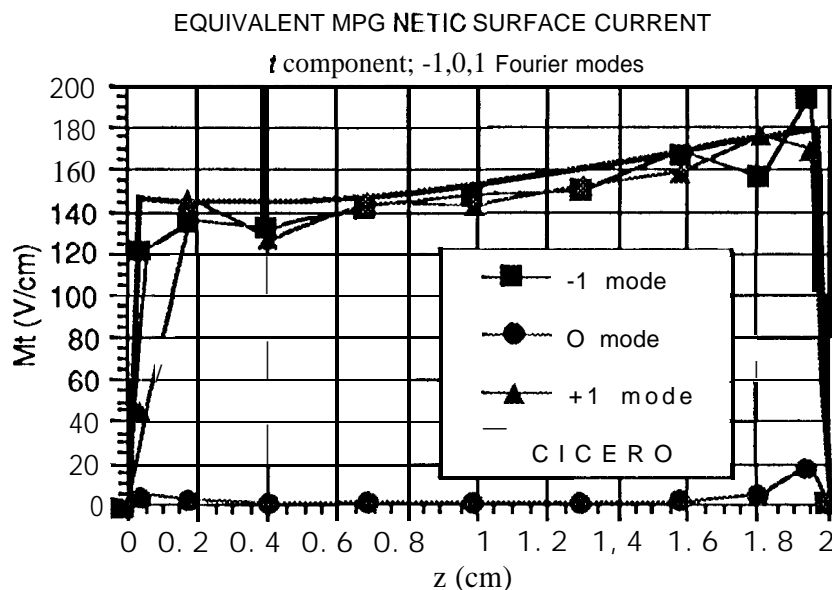
EQUIVALENT MPG NETIC SURFACE CURRENT

t component; -1,0,1 Fourier modes

Figure 2. Current on surface of sphere. The ± 1 modal current should be equal, and identical to the CICERO result. "l'he O modal current should be identically zero,

VI. REFERENCES

[1] W. E. Boyse, A. A. Seidl, "A Hybrid Finite Element Method for Near Bodies of Revolution," *IEEE Trans. Msg.,* Vol. 27, pp. 3833-3836, 1991.

[2] L. N, Medgyesi-Mitschang and J. M. Putnam, "Electromagnetic Scattering from Axially Inhomogeneous Bodies of Revolution," *IEEE Trans. Antennas Propagat.,* Vol. AP-32, pp. 797–806, 1984.